

Problem 6

In the following problems, find the limit of the given sequence as $n \rightarrow \infty$.

$$\frac{n^n}{n!}$$

Solution

Take the limit as $n \rightarrow \infty$.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{n^n}{n!} &= \lim_{n \rightarrow \infty} \frac{\overbrace{n \cdot n \cdot n \cdots n \cdot n \cdot n}^n}{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1} \\ &= \left(\lim_{n \rightarrow \infty} \frac{n}{n} \right) \left(\lim_{n \rightarrow \infty} \frac{n}{n-1} \right) \left(\lim_{n \rightarrow \infty} \frac{n}{n-2} \right) \cdots \left(\lim_{n \rightarrow \infty} \frac{n}{3} \right) \left(\lim_{n \rightarrow \infty} \frac{n}{2} \right) \left(\lim_{n \rightarrow \infty} \frac{n}{1} \right) \\ &= (1)(1)(1) \cdots (\infty)(\infty)(\infty) \\ &= \infty\end{aligned}$$